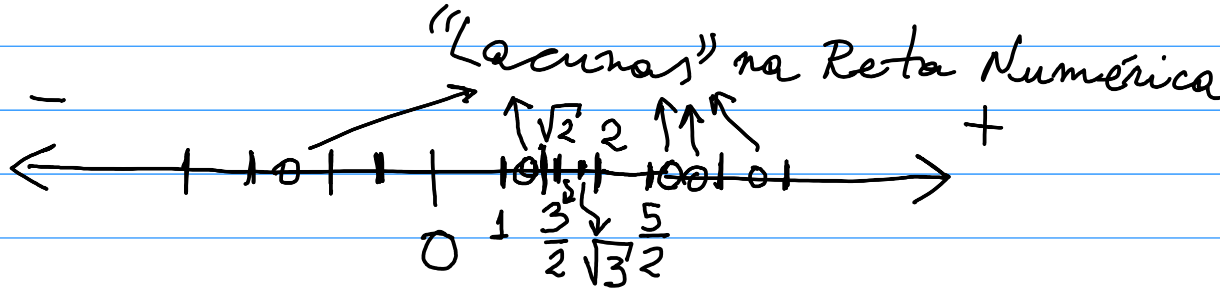
1.5) O Conjunto dos Números Reais ($\mathbb{R}$)



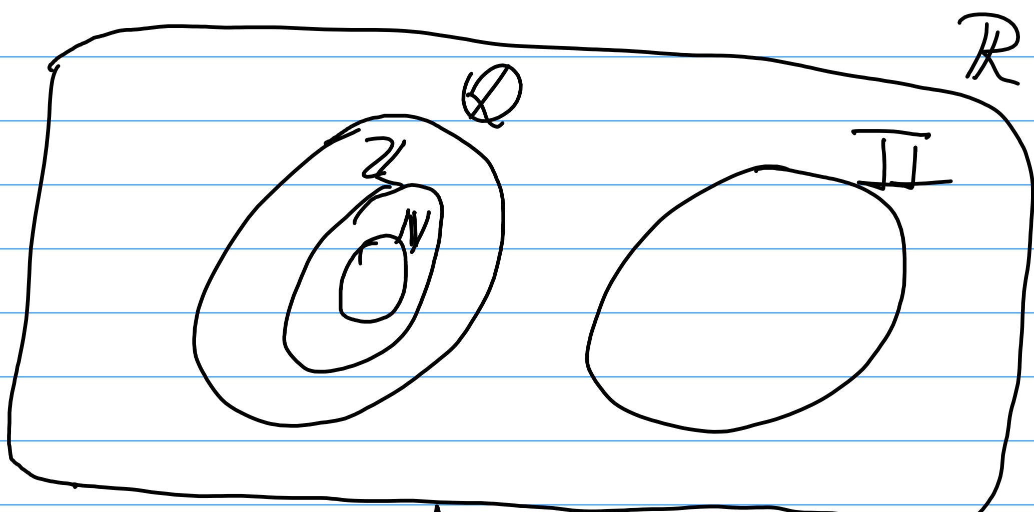
$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \ , \ \mathbb{I}$$

$$\mathbb{Q} \cap \mathbb{I} = \emptyset = \{\ \}$$

Conjunto Vazio

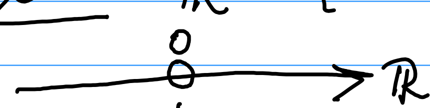
Grosso modo, as "lacunas" na reta numérica são "preenchidas" com a criação do conjunto dos números reais, que pode ser pensado como definido por: $\boxed{\mathbb{R} = \mathbb{Q} \cup \mathbb{I}}$.

Diagrama de Venn



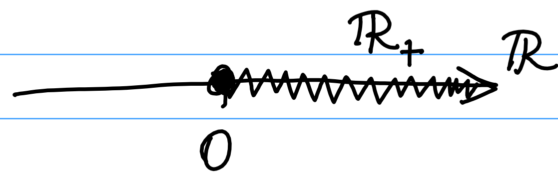
Ex's de Subconjuntos de $\mathbb{R}$

Ex1: Reais Não-Nulos: $\mathbb{R}^* = \{x \in \mathbb{R} \mid x \neq 0\}$



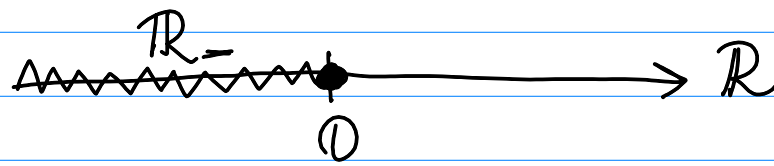
Ex2: Reais Não-Negativos

$$\mathbb{R}_+ = \{x \in \mathbb{R} \mid x \geq 0\}$$



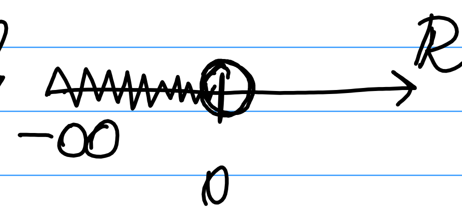
Ex3: Reais Não-Positivos

$$\mathbb{R}_- = \{x \in \mathbb{R} \mid x \leq 0\}$$



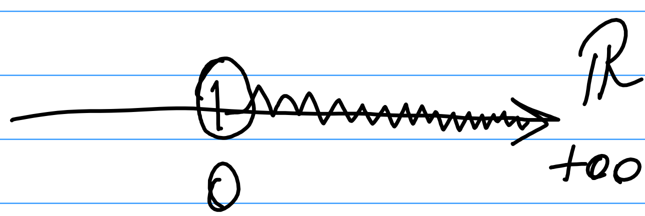
Ex.4: Reais Negativos

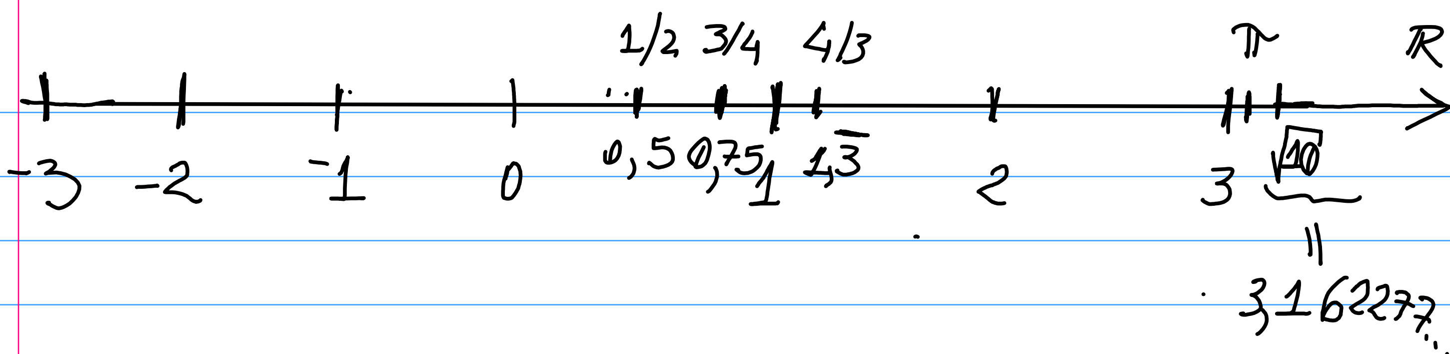
$$\mathbb{R}_-^* = \{x \in \mathbb{R} \mid x < 0\}$$



Ex5: Reais Positivos

$$\mathbb{R}_+^* = \{x \in \mathbb{R} \mid x > 0\}$$





Exercício: Disponha em ordem crescente os nºs $0,\overline{7}$, $0,\overline{71}$, $0,7$, $\frac{3}{4}$, $\frac{\sqrt{2}}{2}$ e $\frac{18}{25}$.

Mais Ex's de Nºs Reais

a) $\frac{4}{0,3}$, $\frac{4}{0,33}$, $\frac{4}{0,333}$, $\frac{4}{0,\overline{3}}$

b) $\frac{2}{\sqrt{2}}$, $\frac{1}{\sqrt{3}}$, $\frac{\sqrt{6}}{4}$, $(1+\sqrt{2})^2$, $(2-\sqrt{2})^2$,

$(3+\sqrt{2})(3-\sqrt{2})$, 2^3, 2^{-4}, $\sqrt{7+4\sqrt{3}}$,

$\frac{\sqrt{2-\sqrt{3}}}{2}$, $\frac{\sqrt{6}-\sqrt{2}}{4}$

Note-se que: $\frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\cancel{(\sqrt{2})^2}}$

$= \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2} \simeq 1,41$

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{(\cancel{\sqrt{3}})^{\cancel{2}}} = \frac{\sqrt{3}}{3}. \checkmark$$

$$(3+\sqrt{2})(3-\sqrt{2}) = 3^2 - (\cancel{\sqrt{2}})^{\cancel{2}} = 9-2=7$$

$$\underbrace{(a+\cancel{b})}\underbrace{(a-\cancel{b})} = a^2 - \cancel{b}^2$$

$$2^3 = \underbrace{2 \times 2 \times 2}_{\text{"3 vezes o 2"}} = 8.$$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{2\times 2\times 2\times 2} = \frac{1}{16}$$

ou

$$\frac{1}{2^3 . 2} = \frac{1}{16}$$

Definições

$$a^n = \underbrace{a \times a \times \ldots \times a}_{\text{"n vezes"}}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\boxed{a^{m+n} = a^m . a^n}$$

<u>Ex.'s:</u> $2^4 = 2^{3+1} = 2^3 . 2 = 16$

$$2^{10} = 2^6 . 2^4$$
$$2^{10} = 2^4 . 2^2 . 2^4$$
$$2^{10} = 16.4.16$$
$$2^{10} = 1024.$$

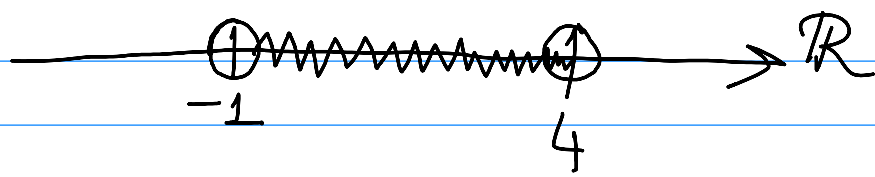
Intervalos Reais

I.1) Intervalo Aberto de Extremos a e b

$$]a, b[\text{ ou } (a, b)$$

$$]a, b[= \{x \in \mathbb{R} \mid a < x < b\}$$

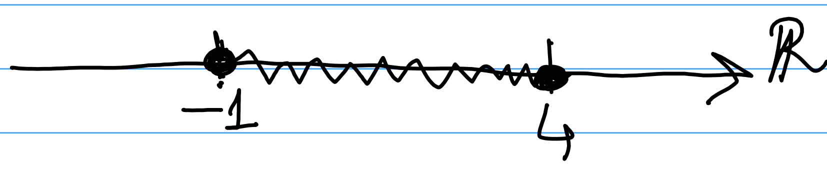
Ex: $]-1, 4[= \{x \in \mathbb{R} \mid -1 < x < 4\}$



I.2) Intervalo Fechado de Extremos a e b

$$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$$

Ex: $[-1, 4] = \{x \in \mathbb{R} \mid -1 \leq x \leq 4\}$

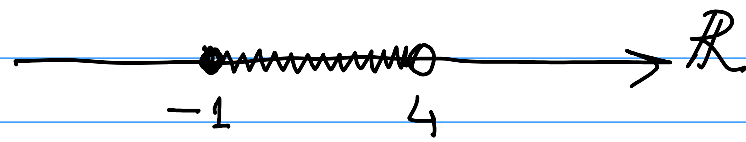


I.3) Intervalo Aberto à Direita (ou Fechado à Esquerda) de Extremos a e b

$[a, b[$ ou $[a, b)$

$$[a, b[= \{x \in \mathbb{R} \mid a \leq x < b\}$$

Ex: $[-1, 4[= \{x \in \mathbb{R} \mid -1 \leq x < 4\}$

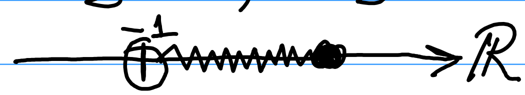


I.4) Intervalo Aberto à Esquerda (ou fechado à Direita) de Extremos a e b

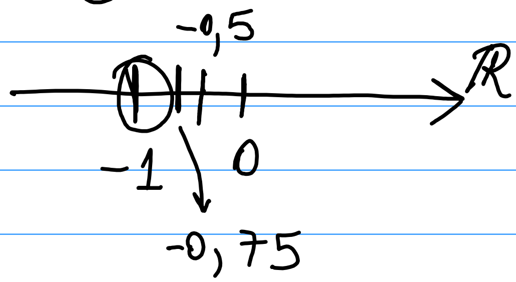
$]a, b]$ ou $(a, b]$

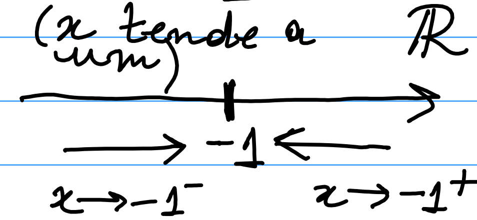
$$]a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$$

Ex: $]-1, 3] = \{x \in \mathbb{R} \mid -1 < x \leq 3\}$

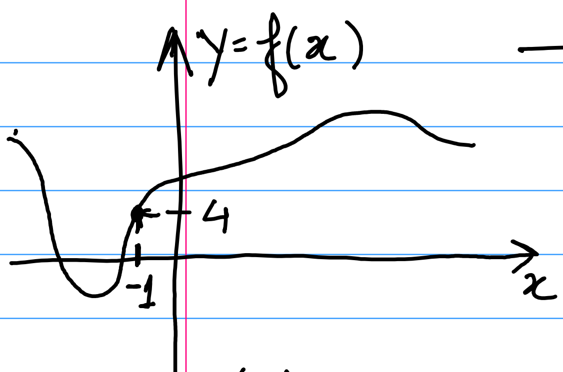


$x \longrightarrow 1$
(x tende a um)





$\boxed{x \longrightarrow -1}$



$-0,999999999999$

(x tende a -1, pela direita e pela esquerda) $x \longrightarrow -1^+$, $x \longrightarrow -1^-$

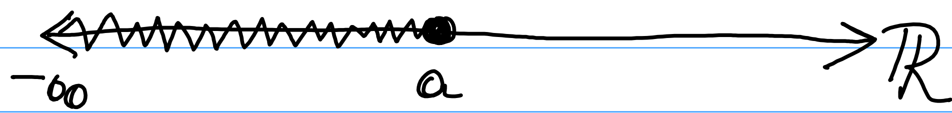
$$\boxed{\lim_{x \to -1} f(x) = 4}$$

(Um Pouco da Ideia de Limite de uma função, quando x tende a -1 $(x \to -1)$)

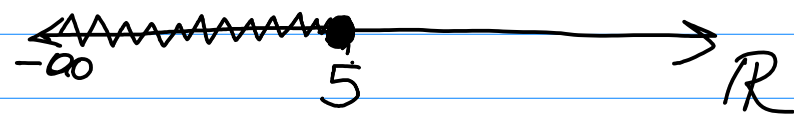
I.5) Intervalos Infinitos

I.5.1) $]-\infty, a]$ ou $(-\infty, a]$

$$]-\infty, a] = \{x \in \mathbb{R} \mid \underline{\underline{x \leq a}}\}$$

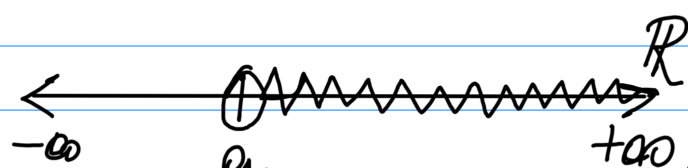


$\underline{Ex}$: $]-\infty, 5] = \{x \in \mathbb{R} \mid x \leq 5\}$

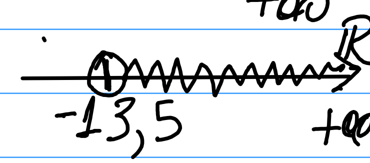


I.5.2) $]a, +\infty[$ ou $(a, +\infty)$

$(a, +\infty) = \{x \in \mathbb{R} \mid x > a\}$

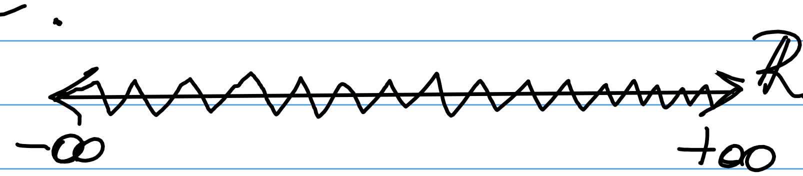


$\underline{Ex}$: $(-13,5, +\infty) = \{x \in \mathbb{R} \mid x > -13,5\}$

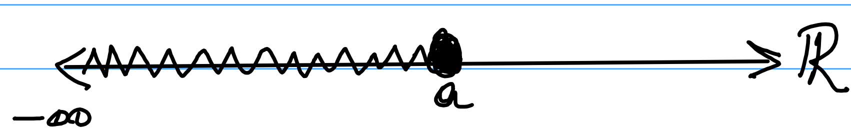


Caso Particular:

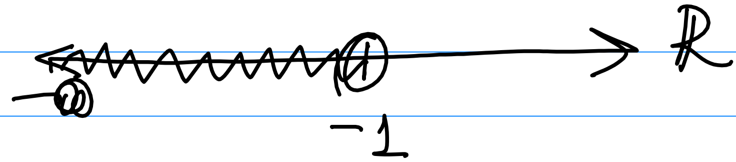
$(-\infty, +\infty) = \mathbb{R}$



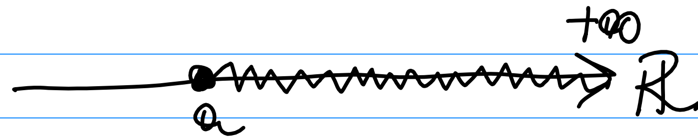
I.5.3) $]-\infty, a[= \{x \in \mathbb{R} \mid \underline{\underline{x < a}}\}$



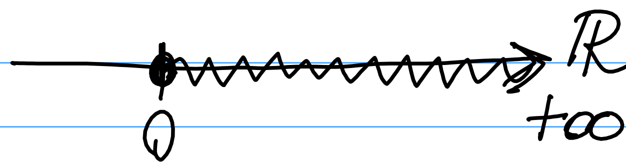
Ex: $]-\infty, -1[\; = \{x \in \mathbb{R} \mid x < -1\}$



I.5.4) $[a, +\infty[\; = \{x \in \mathbb{R} \mid x \geq a\}$



Exemplo: $[0, +\infty) = \{x \in \mathbb{R} \mid \underline{x \geq 0}\}$



⚠ O domínio da função $f(x) = \sqrt{x}$ é $(0, +\infty)$.

$\sqrt{-4}, \sqrt{-1}, \sqrt{-2}$

$f(0) = \sqrt{0} = 0$

$f(1) = \sqrt{1} = 1$

$f(2) = \sqrt{2} \cong 1,41$

$f(3) = \sqrt{3} \cong 1,73$

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi}$$

$\boxed{i = \sqrt{-1}}$

$p^+ \overset{r}{\text{—}} e^-$ (Átomo de H)

$\nexists \sqrt{-4}$, $\exists \sqrt{4} = 2$

$V(r) = \frac{-ke^2}{r}$

Níveis de energia do elétron: $\boxed{E_n \propto \frac{-1}{n^2}}$

$\boxed{x < 0, \nexists \sqrt{x}}$